How to Optimize the *Shape of Quantum Light* in Quantum Metrology?

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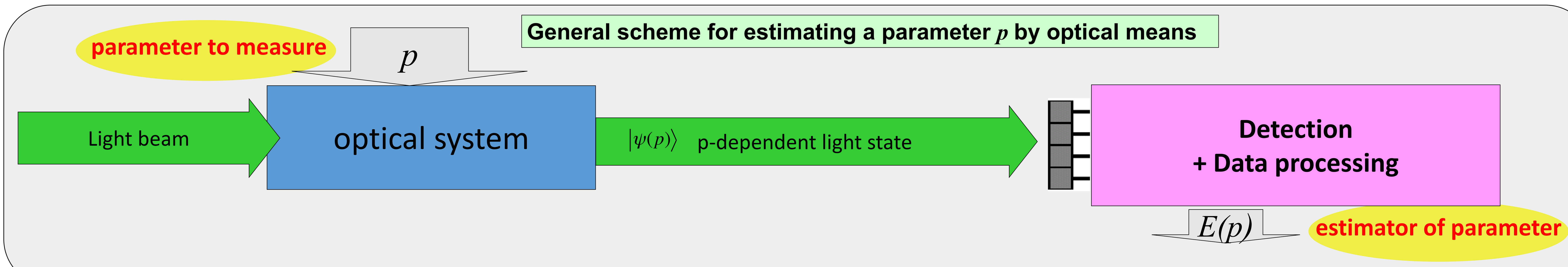
Because of the 1/N^x quantum limits in optical measurements (x=0,5 for standard quantum noise and 1 for Heisenberg limited measurements), **the best strategy for maximum sensitivity is to use states of light with very high photon number N**. In this respect, **multimode Gaussian states of light**, encompassing intense coherent states, squeezed states and EPR entangled states, are the best practical choice. This strategy has indeed been successfully implemented to reduce the quantum noise floor in the gravitational wave interferometers which use ultra-intense lasers and vacuum squeezed state. We have generalized this approach to any parameter estimation by optical means, and found the expression of the quantum Cramer Rao limit when one uses multimode non-classical Gaussian states, **with the possibility of optimizing not only the multimode Gaussian quantum state, but also *the shape of the modes* in which the state "lives".**

We have identified in particular a **"noise mode",** the quantum fluctuations of which are responsible for the noise in the estimation, and given techniques enabling us to reach the quantum Cramer-Rao limit. We have implemented this approach and improved parameter estimation beyond the standard quantum noise in the case of measurements of frequency shifts and beam displacements.

General scheme for estimating a parameter p by optical means



our choice of quantum states carrying the information about the parameter p : the multimode Gaussian pure states

includes **a wide class of non-classical states**
- coherent state
- multimode squeezed state
- Einstein Podolsky Rosen entangled state
- choice left also for number and spatio-temporal shape of modes

in mW beams photon number N can be as large as 10^{15}

Fock states, NOON states … are not included, as they are so far produced only with low N values

Ultimate quantum limit (Quantum Cramer Rao limit) for the measurement of p with Gaussian states:

$$\Delta p_{Cramer-Rao} = \frac{p_c}{2\sqrt{N}} \Delta E_{detection}$$

p-sensitivity

« shot noise »

Quantum fluctuations in a single mode, the **detection mode**

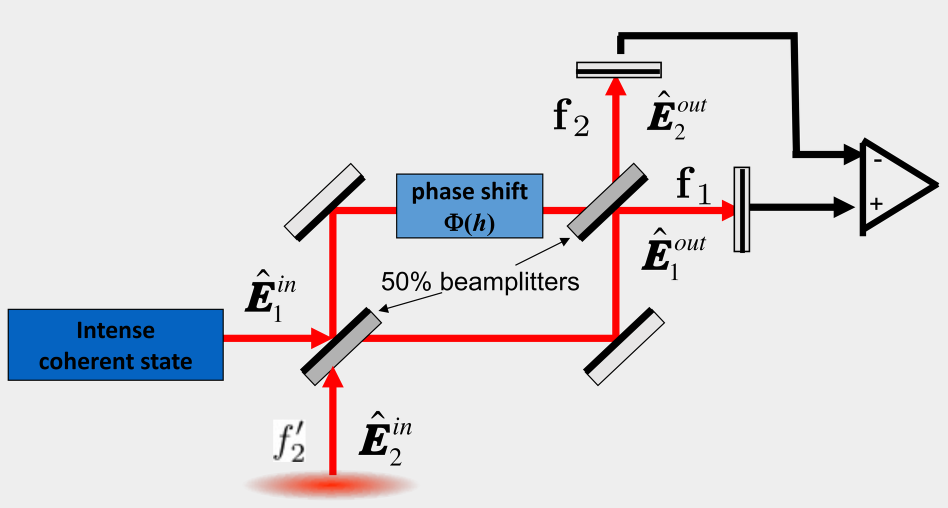
$$u_{detection} = p_c \frac{\partial < mean\,field >}{\partial p}$$

value independent of the fluctuations of all other modes

Conclusions for the experimentalist

To get the lowest possible Quantum Limit
- **Put maximum power in coherent state**
- **Use squeezed light in only one mode, but in the right one!** (the detection mode)
- **Do not use an entangled state** (at the detection stage)

Application: interferometric measurement of phase



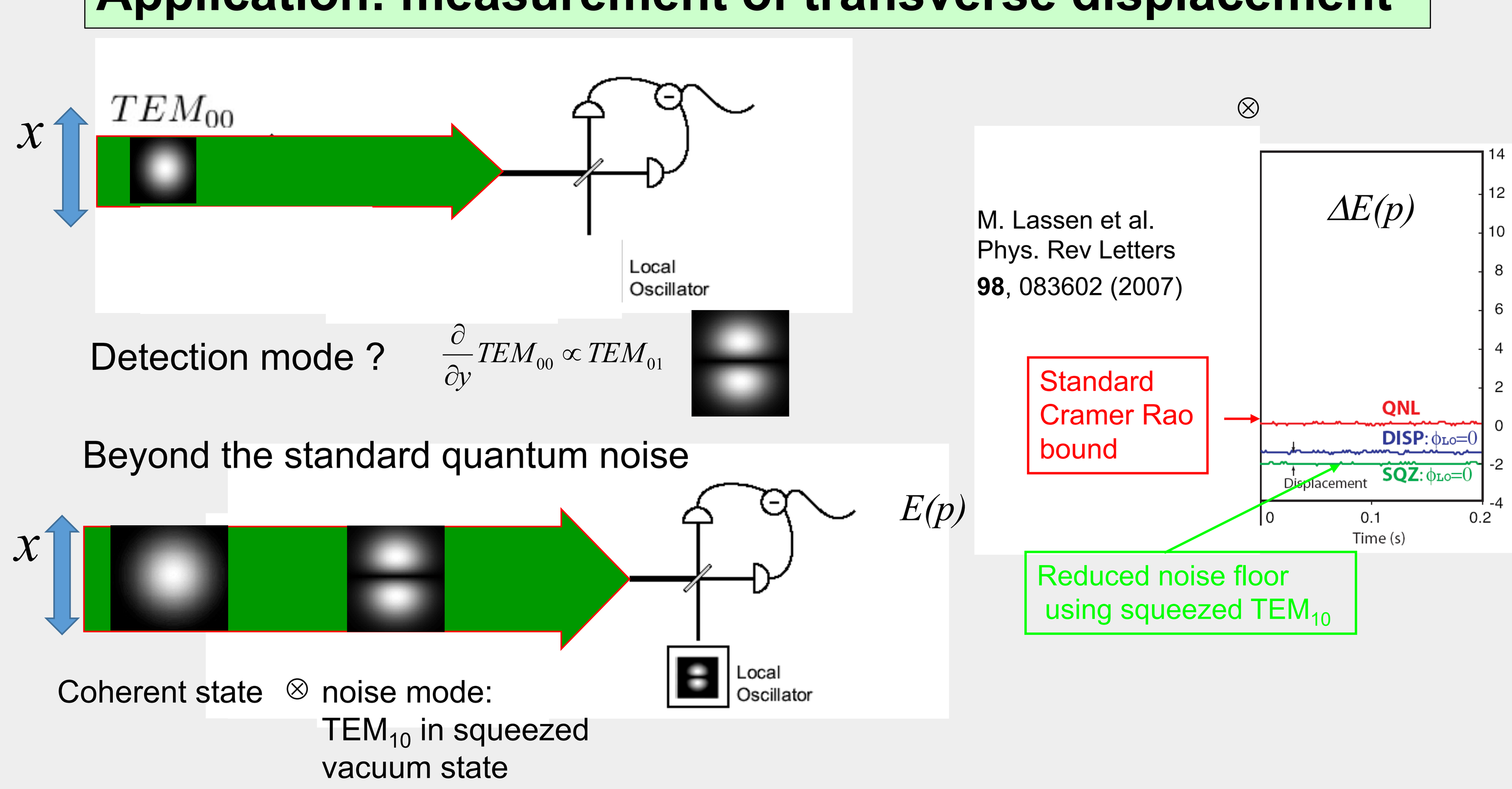
detection mode at the output of interferometer: $\frac{1}{\sqrt{2}}(\mathbf{f}_1 - \mathbf{f}_2)$

detection mode at the input: f'_2

One retrieves **Caves configuration**:
-an intense coherent state on one input port
- a vacuum squeezed state on the other.

implemented now on gravitational interferometers

Application: measurement of transverse displacement



also implemented in ranging and measurement of frequency shifts
can also improve clock synchronization

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