Bureau International des Poids et Mesures

On the possible future revision of the SI

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and

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BIPM

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International Symposium on Measurement Technology and Intelligent Instruments

Bureau International des Poids et Mesures

BIPM

- Based in Paris and financed by 55 Member States and 37 Associate States/Economies.
- Maintains scientific laboratories in areas of: mass, time, electricity, ionizing radiation, and chemistry.

CIPM

- Made up of eighteen individuals, from Member States.
- Meets annually to promote worldwide uniformity in units of measurement.
- Is the management board for the BIPM

CGPM

- Made up of representatives from Member States.
- Meets in Paris every four years to discuss the status of international metrology.

The International System of Units (SI)

Prefixes

Table 5. SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	µ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Base units

Table 1. SI base units

Base quantity		SI base unit	
Name	Symbol	Name	Symbol
length	l, x, r, etc.	metre	m
mass	m	kilogram	kg
time, duration	t	second	s
electric current	I, i	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	I_v	candela	cd

Derived units

Table 3. Coherent derived units in the SI with special names and symbols

	SI coherent derived unit [(a)]			
Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
plane angle	radian [(b)]	rad	1 [(b)]	m/m
solid angle	steradian [(b)]	sr [(c)]	1 [(b)]	m^2/m^2
frequency	hertz [(d)]	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, amount of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiant flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge, amount of electricity	coulomb	C		s A
electric potential difference, electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}\ A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Celsius temperature	degree Celsius [(e)]	°C		K
luminous flux	lumen	lm	cd sr [(c)]	cd
illuminance	lux	lx	lm/m^2	m^{-2} cd
activity referred to a radionuclide [(f)]	becquerel [(d)]	Bq		s^{-1}
absorbed dose, specific energy (imparted), kerma	gray	Gy	J/kg	$m^2\ s^{-2}$
dose equivalent, ambient dose equivalent, directional dose equivalent, personal dose equivalent	sievert [(g)]	Sv	J/kg	$m^2\ s^{-2}$
catalytic activity	katal	kat		s^{-1} mol

The 8th edition of the SI Brochure is available from the BIPM website.

The base units of the SI

3 definitions based on fundamental (or conventional) constants:

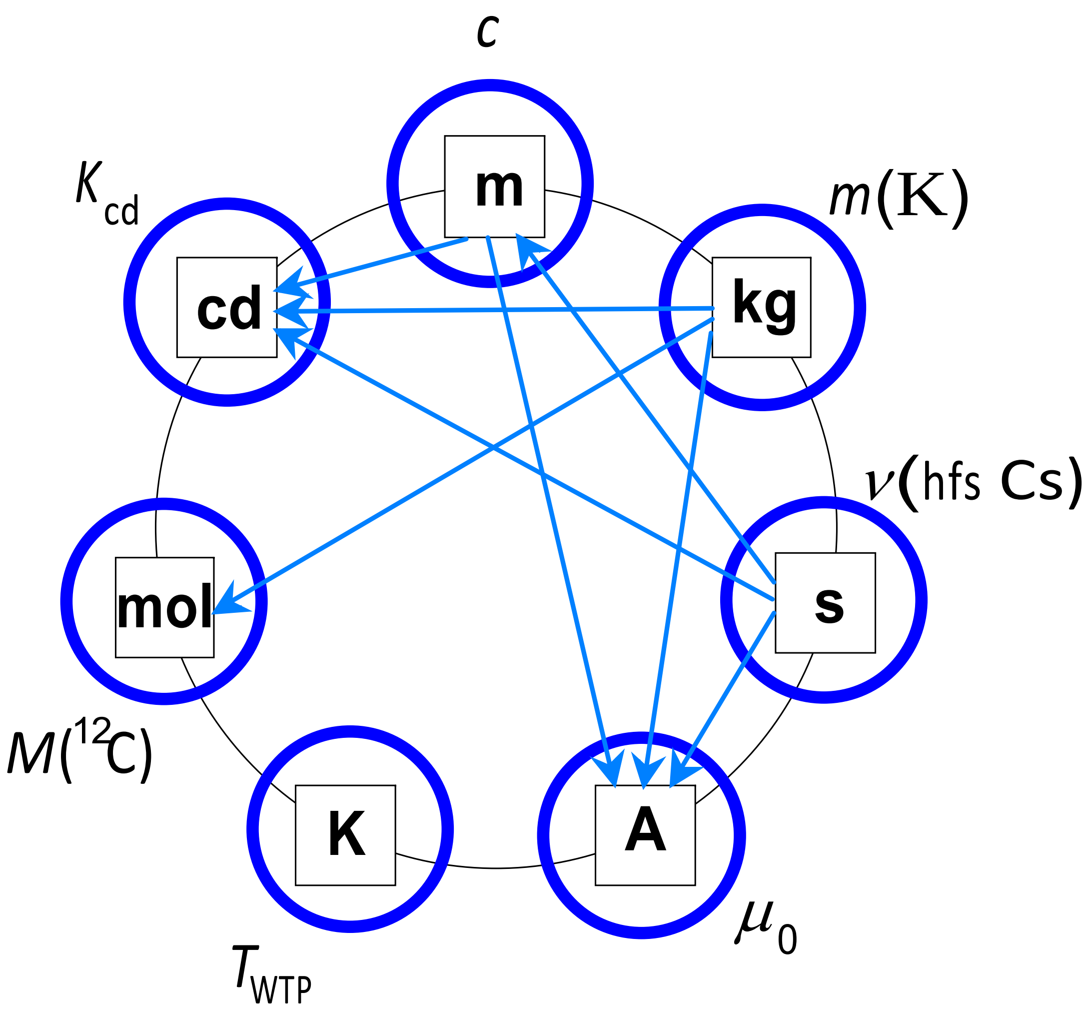
- **metre (c)**
- **ampere (μ_0)**
- **candela (K_{cd})**

3 definitions based on material properties:

- **second (^{133}Cs)**
- **kelvin (H_2O)**
- **mole (^{12}C)**

1 definition based on an artefact:

- **kilogram (IPK)**



On the possible future revision of the SI

- **Why change?**
  - Mass (kilogram)
  - Electricity (ampere)
  - Temperature (kelvin)
  - Amount of substance (mole)
- What happens next?

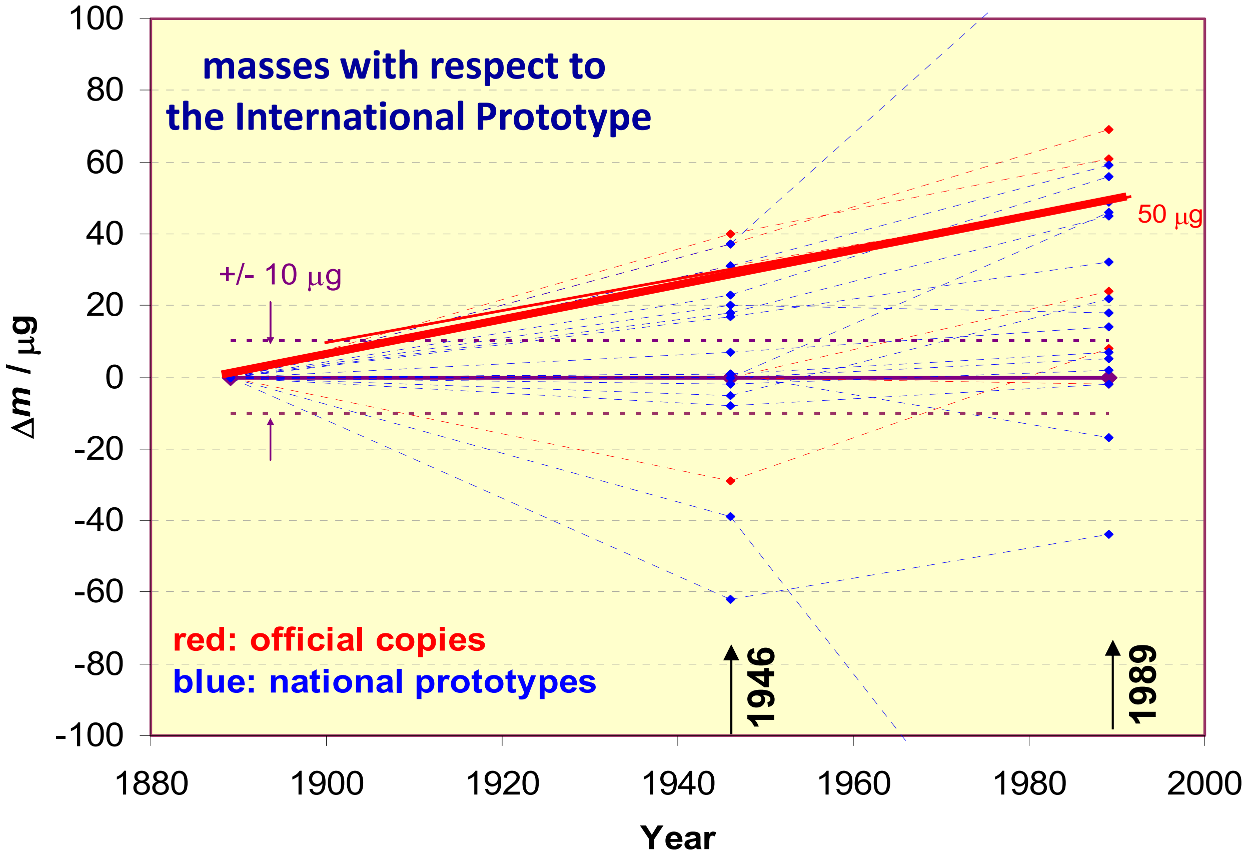
The definition of the kilogram in the SI

The kilogram is the unit of mass - it is equal to the mass of the international prototype of the kilogram.

- manufactured around 1880 and ratified in 1889
- represents the mass of 1 dm^3 of H_2O at its maximum density (4 °C)
- alloy of 90% Pt and 10% Ir
- cylindrical shape, $Ø = h \sim 39$ mm
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.

Calibration history of the oldest national prototypes



Variations of about **50 µg (5×10^{-8})** in the mass of the standards **over 100 years**

Approximately **0.5 µg / year**

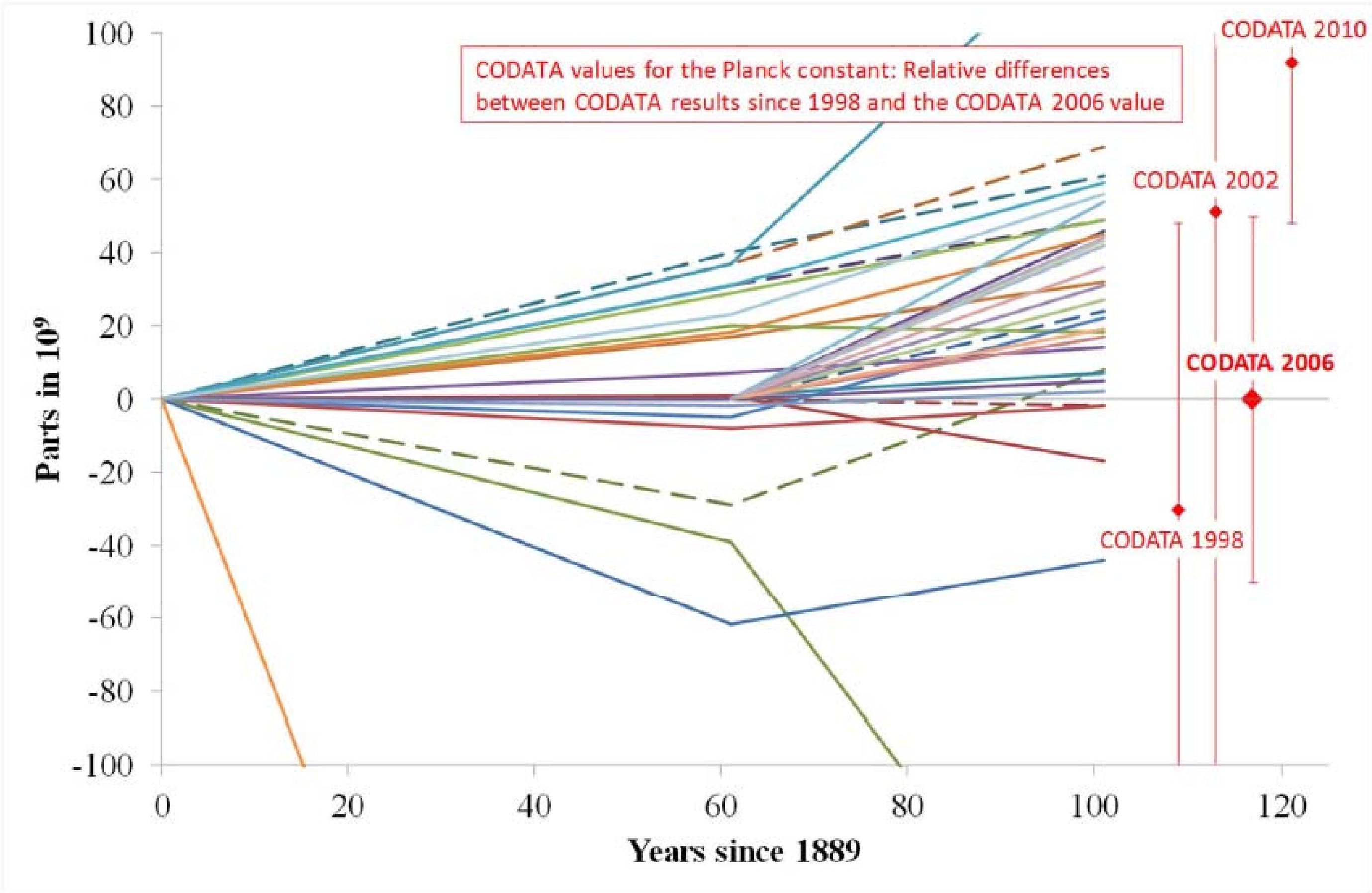
Masses can now be compared to within 1 µg

A drifing kg also influences the electrical units

Is the IPK losing mass? or, are the other standards gaining mass?

If we could answer this question, we would have a better definition!

Long-term stability

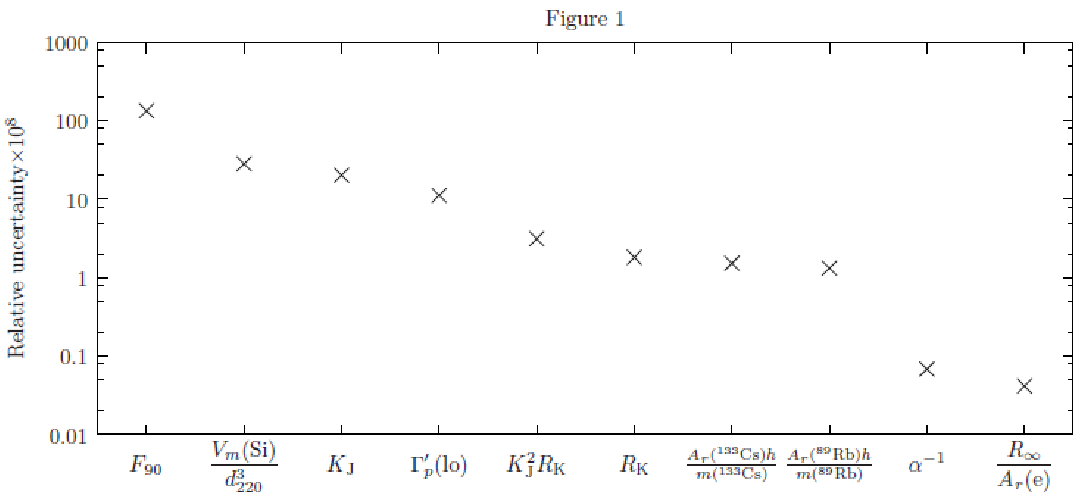


What is the motivation for any new definitions?

Adopt a system that does not rely on any artefacts. Based on fixed values of selected constants.

1. Solve the “kg problem”.
2. Bring the electrical units back into the SI
3. Reduce the uncertainty of certain fundamental constants

> “If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.”
>
> James Clerk Maxwell, 1870



Milton *et al* 2010

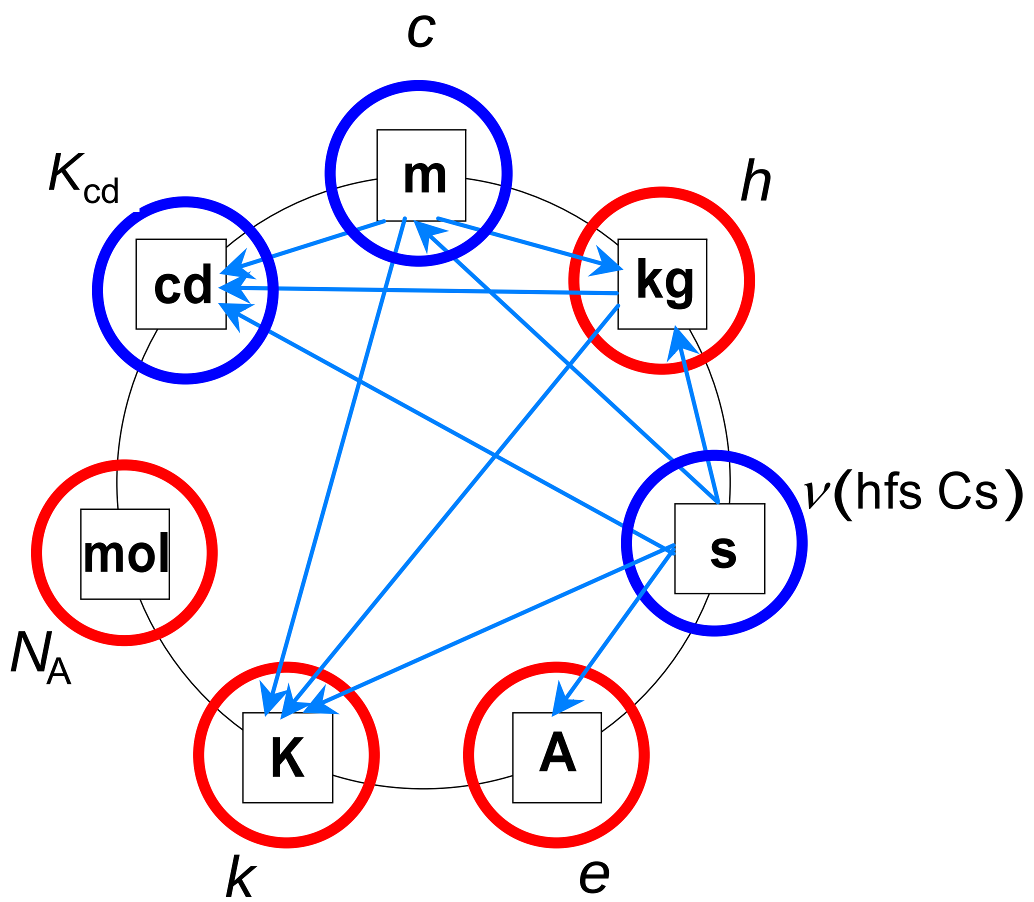
Proposal for a new SI, with 4 new definitions

Definitions based on fundamental (or conventional) constants:

- metre (*c*)
- kilogram (*h*)
- ampere (*e*)
- candela (K_{cd})
- mole (N_A)
- kelvin (*k*)

Definition based on material property:

- second (^{133}Cs)



(I. Mills et al., *Metrologia*, 2006, 43, 227-246)

Proposed new definition for the kg

The kilogram, unit of mass, is such that the Planck constant *h* is exactly equal to 6.626 068 XX $\times 10^{-34}$ joule second:

$$h = 6.626\,068\,XX \times 10^{-34}\ \mathrm{kg\ m^2\ s^{-1}}$$

The value of *h* is fixed by nature

The numerical value of h is fixed by the definition of the kg

The effect of this equation is to define 1 kg

The units m and s are defined in the SI

How would this work in practice?

- **The watt balance equates electrical and mechanical power**
  - electrical power can be expressed in terms of *h* using the Josephson and quantum Hall effects
- **The "Avogadro" Experiment determines the mass of a single ^{28}Si atom**
  - m_u can be expressed in terms of *h* using extremely accurate measurements of the Rydberg constant.

Proposed new definition for the ampere

The ampere, unit of electric current, is such that the elementary charge is exactly $1.60217653 \times 10^{-19}$ coulomb.

How would this work in practice?

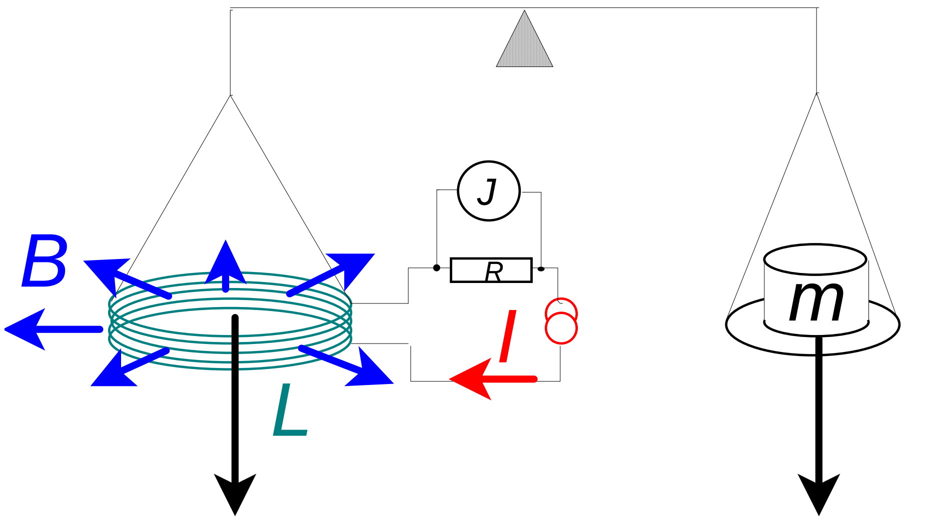
- **The quantum Hall effect defines an impedence in terms of h/e^2**
- **The Josephson effects defines a voltage in terms of $2e/h$**
- **If h is fixed by the definition of the kilogram and e by the definition of the ampere, then we also have an impedence and a voltage standard.**

On the possible future revision of the SI

- Why change?
  - **Mass (kilogram)**
  - Electricity (ampere)
  - Temperature (kelvin)
  - Amount of substance (mole)
- What happens next?

Watt balance principle - 1

Phase 1: static experiment
(weighing mode)



$$F_{el} = I\,L\,B$$

$$F_m = m\,g$$

$$mg = -I\,\frac{d\Phi}{dz}$$

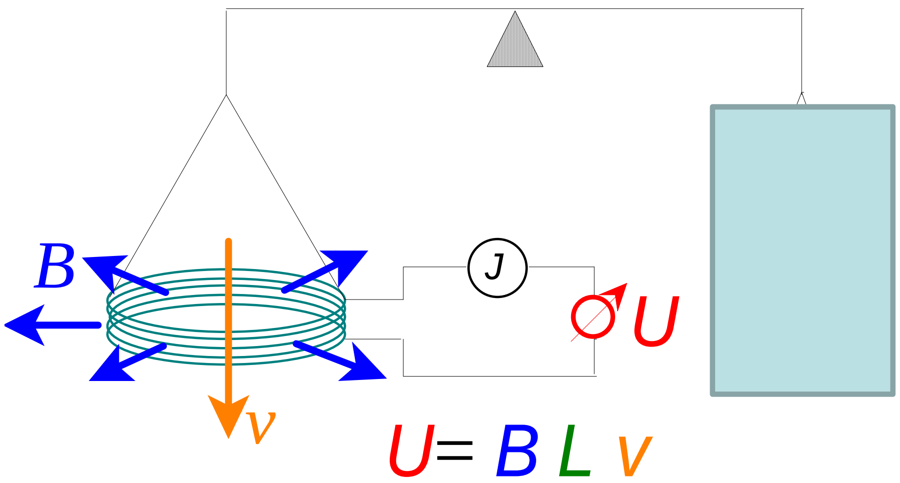
In a radial magnetic field, this can be simplified to

$$m\,g = I\,L\,B$$

current → I; wire length → L; flux density → B

Watt balance principle - 2

Phase 2: dynamic experiment
(moving mode)



Coil is moved through the magnetic field and a voltage is induced.

$$U = -\frac{d\Phi}{dt} = -v\frac{d\Phi}{dz}$$

In a radial magnetic field, this can be simplified to

$$U = B L v$$

ind. voltage

flux density

wire length

velocity

Watt balance equations

Static phase: $$m\,g = I\,B\,L$$

Dynamic phase: $$U = v\,B\,L$$

If the coil and the field are constant: $$U\,I = m\,g\,v \qquad \cancel{B\,L}$$

$$P_{\mathrm{el}} = P_{\mathrm{mech}}$$

Note

the watt balance does not realize a direct conversion of electrical to mechanical energy.

Link between the kg and the Planck constant

U and *R* are measured using the Josephson effect and the quantum Hall effect.

$$U\ I = \frac{U_1\, U_2}{R} = C_{\mathrm{el}}\ f_1\ f_2\ h$$

$$U = u\,\frac{n\ f}{K_J} = u\,\frac{n\ f}{2\,e}\,h$$

$$R = r\,\frac{R_K}{i} = r\,\frac{h}{i\,e^2}$$

⇩

$$U\,I = m\,g\,v$$

⇨

$$h = \frac{m\,g\,v}{C_{\mathrm{el}}\ f_1\ f_2}$$

A new definition of the kg requires the measurement of *h* with an uncertainty of some parts in 10^8.

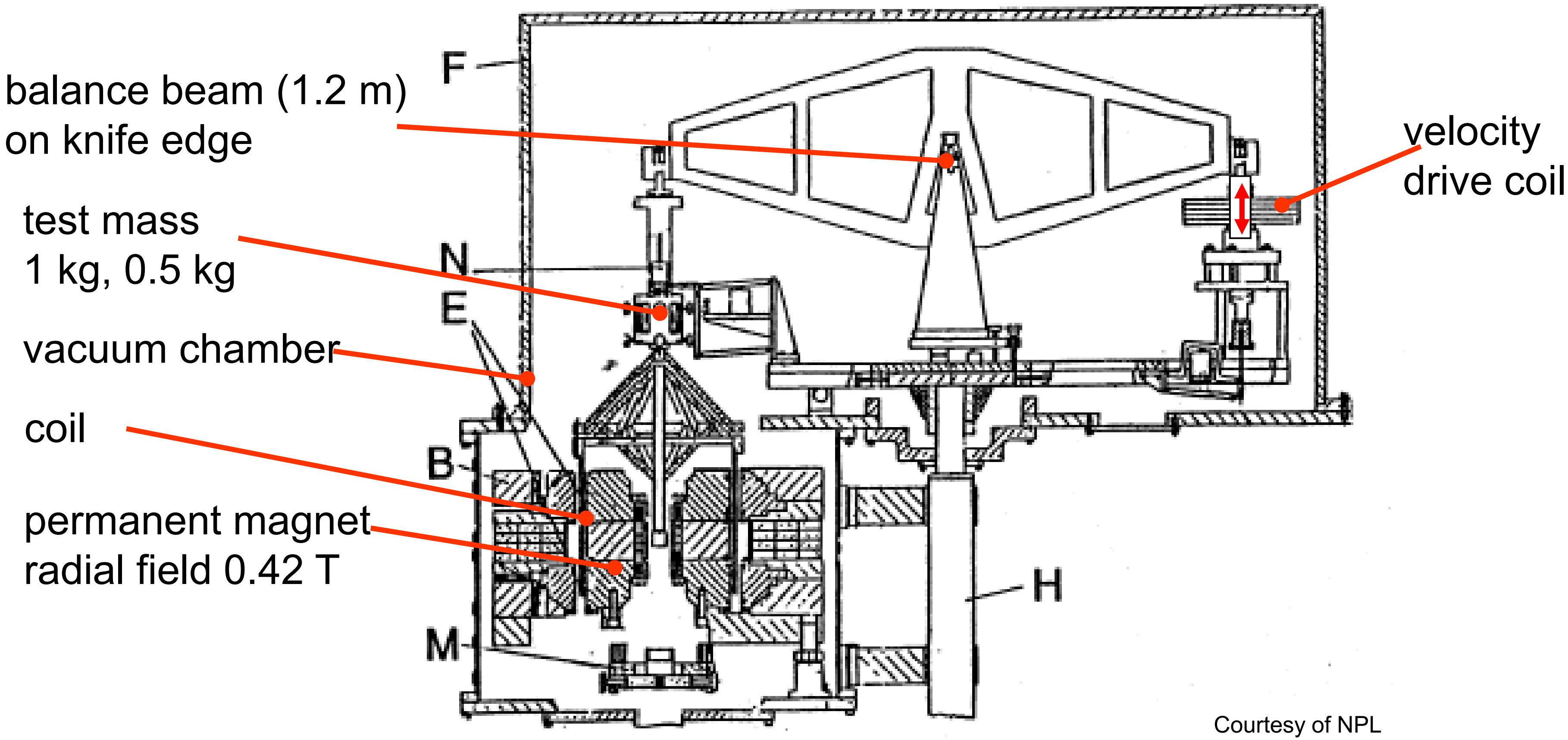
Watt balance experiments

NPL, 1976	**"first watt balance"**
NIST, 1980	"**biggest watt balance**", with superconducting magnet ► **NIST-4**
METAS, 1997	"**smallest watt balance**" ► **BWM II**
LNE, 2001	"**moving beam watt balance**"
BIPM, 2003	"**single mode watt balance**"
NIM, 2006	"**mutual inductance <u>joule</u> balance**"
NRC, 2009	
MSL, 2009	"**oscillatory watt balance/pressure balance**"
KRISS, 2012	

The NPL watt balance – Mark II

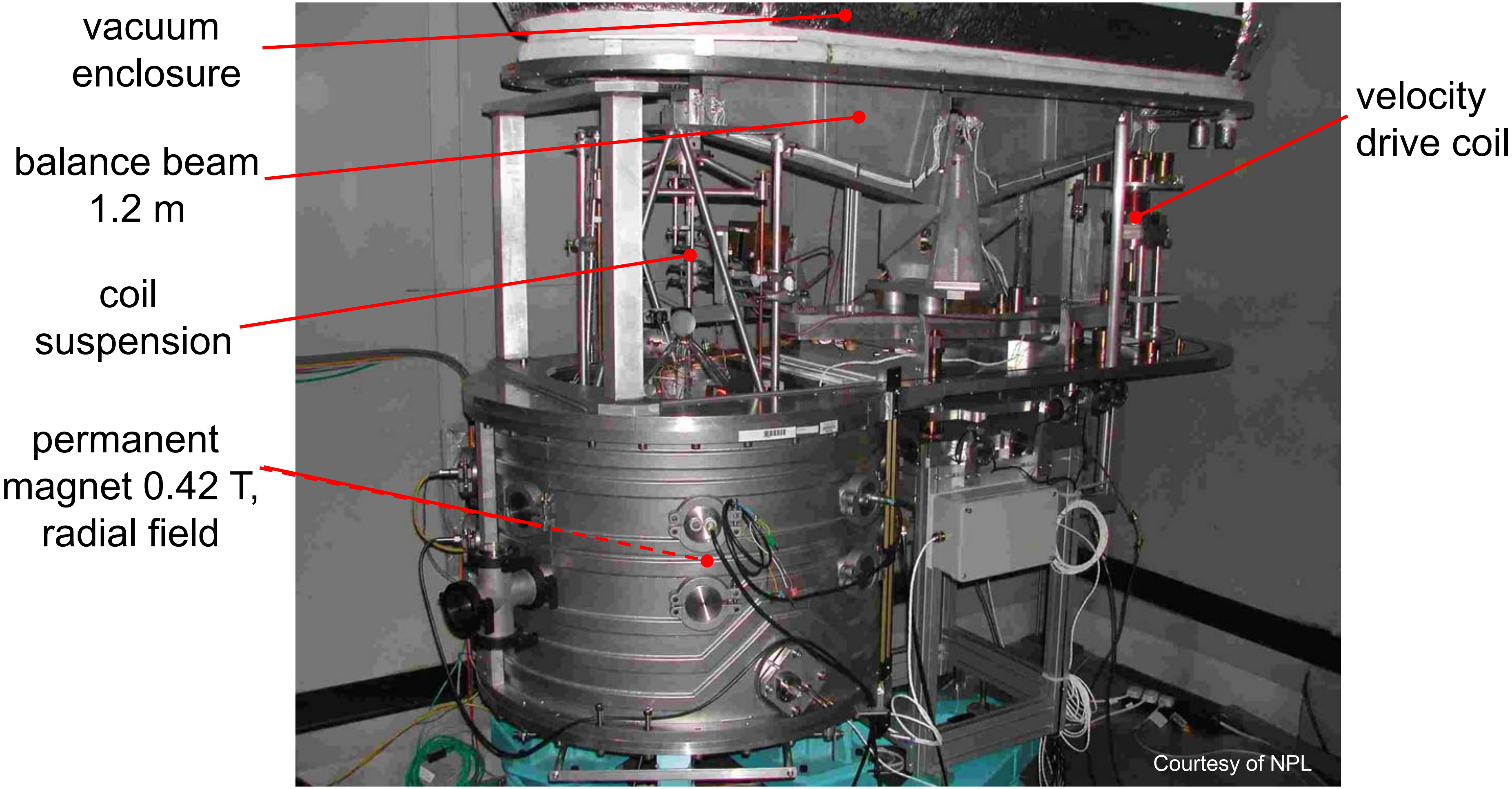
Watt balance principle was proposed by B. Kibble, NPL in 1976

Work on Mark II started around 1990



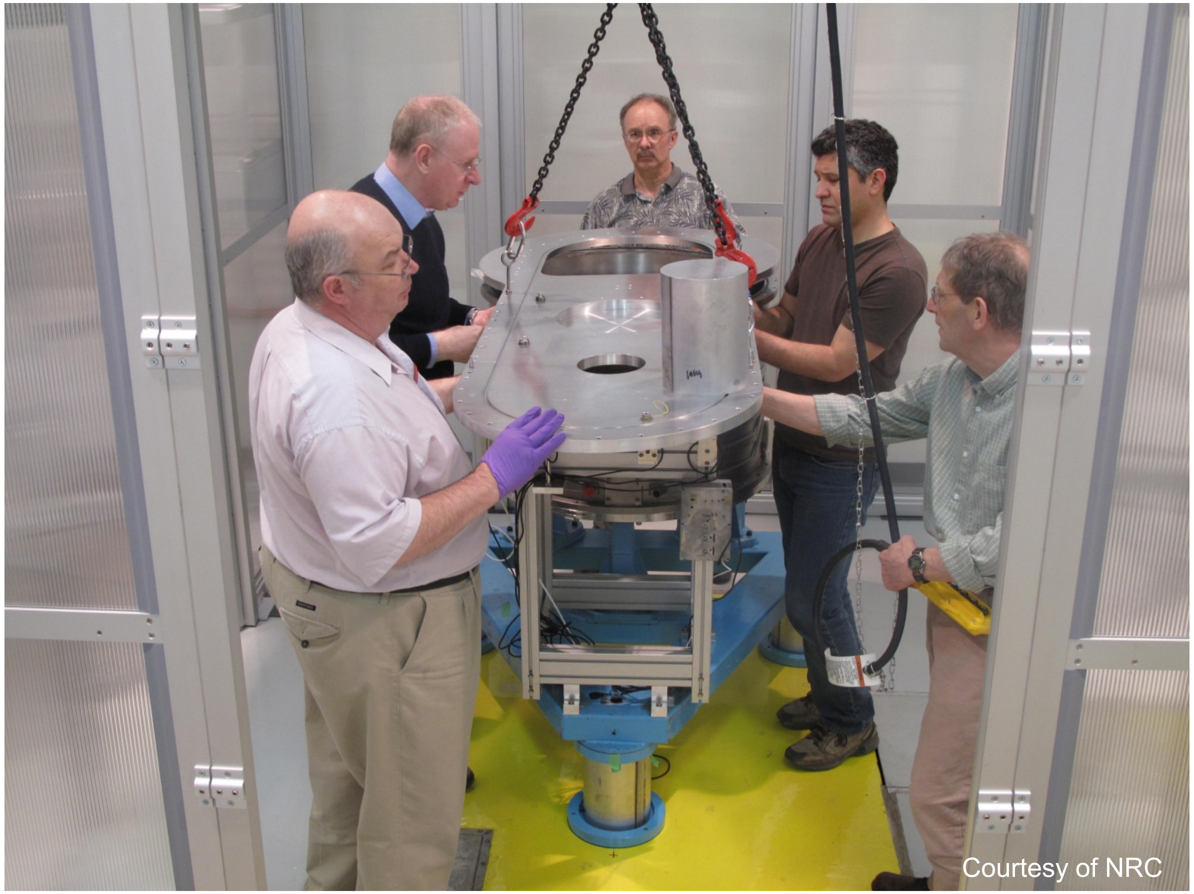
Courtesy of NPL

The NPL watt balance – Mark II



NPL watt balance, starting a new life as the NRC watt balance

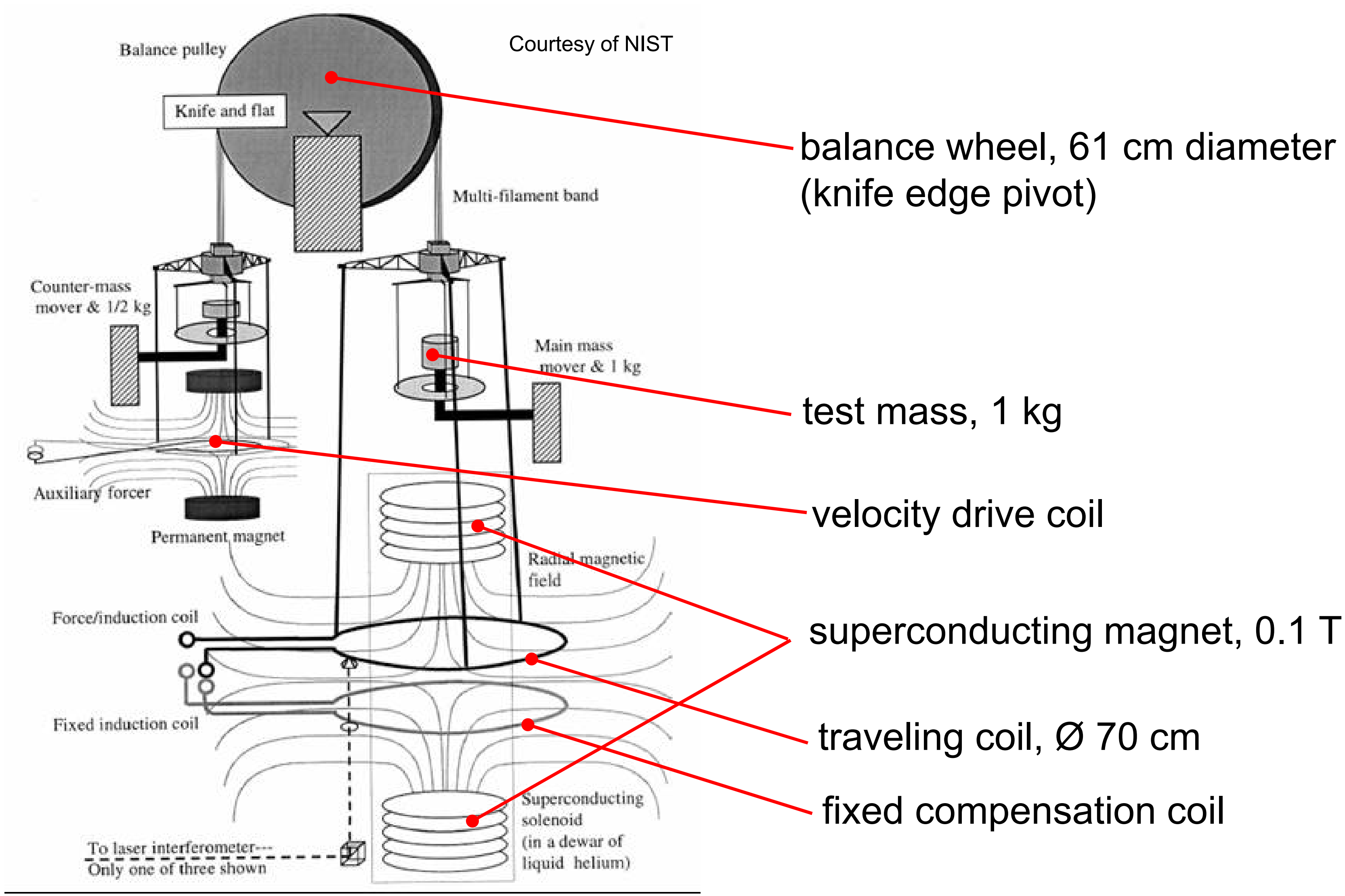
Shipped in summer 2009 from NPL, Teddington, to NRC, Ottawa



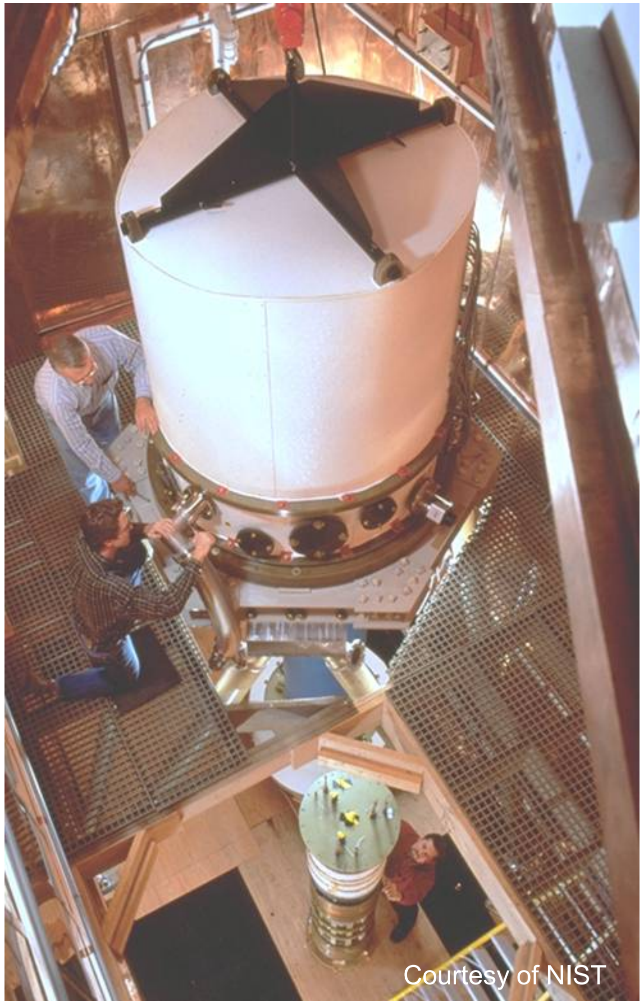
u_r (h, 2012) = 6.5 x 10^{-8}

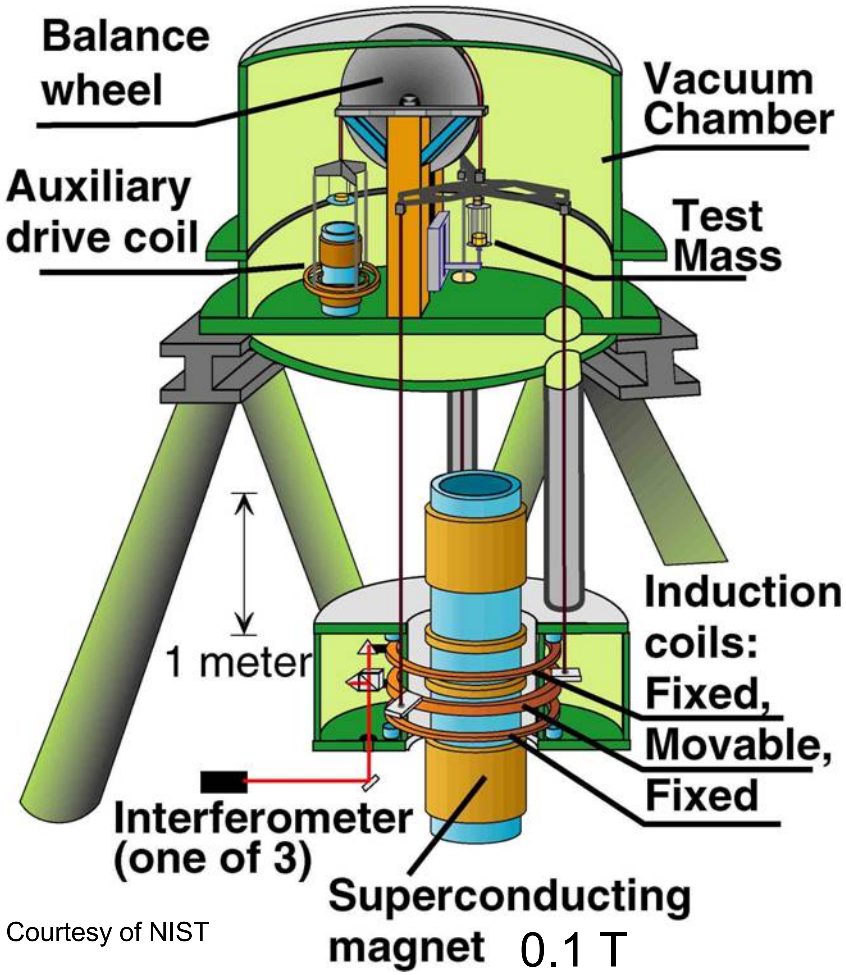
A G Steele *et al* 2012 *Metrologia* **49** L8

The NIST watt balance – started 1980



The NIST watt balance



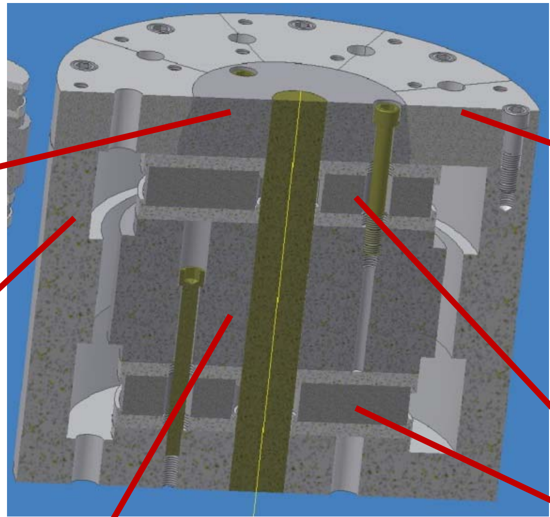


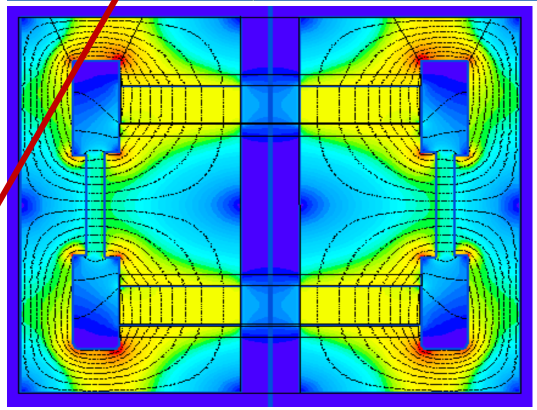
Courtesy of NIST

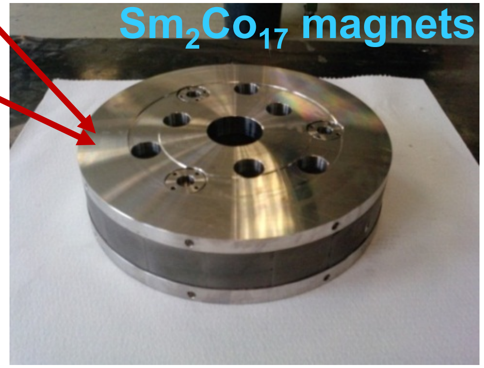
0.1 T

u_r (h, 1998) = 9 x 10^{-8} u_r (h, 2005) = 5 x 10^{-8} u_r (h, 2007) = 3.6 x 10^{-8}

BIPM watt balance: magnet fabrication



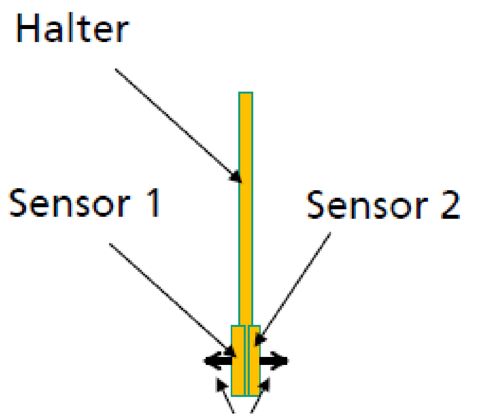




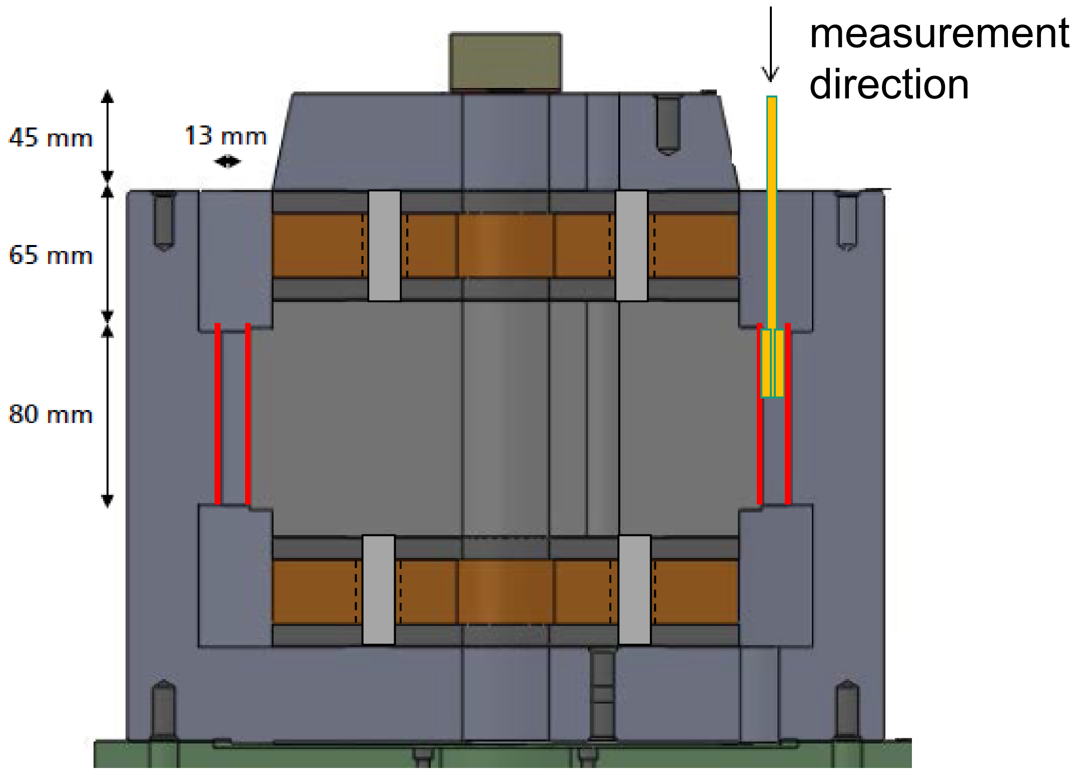
- All components meet the specifications
- Trial assembly (without the magnets) successful, uniformity of the gap width verified (<1 μm)

BIPM watt balance: magnet verification

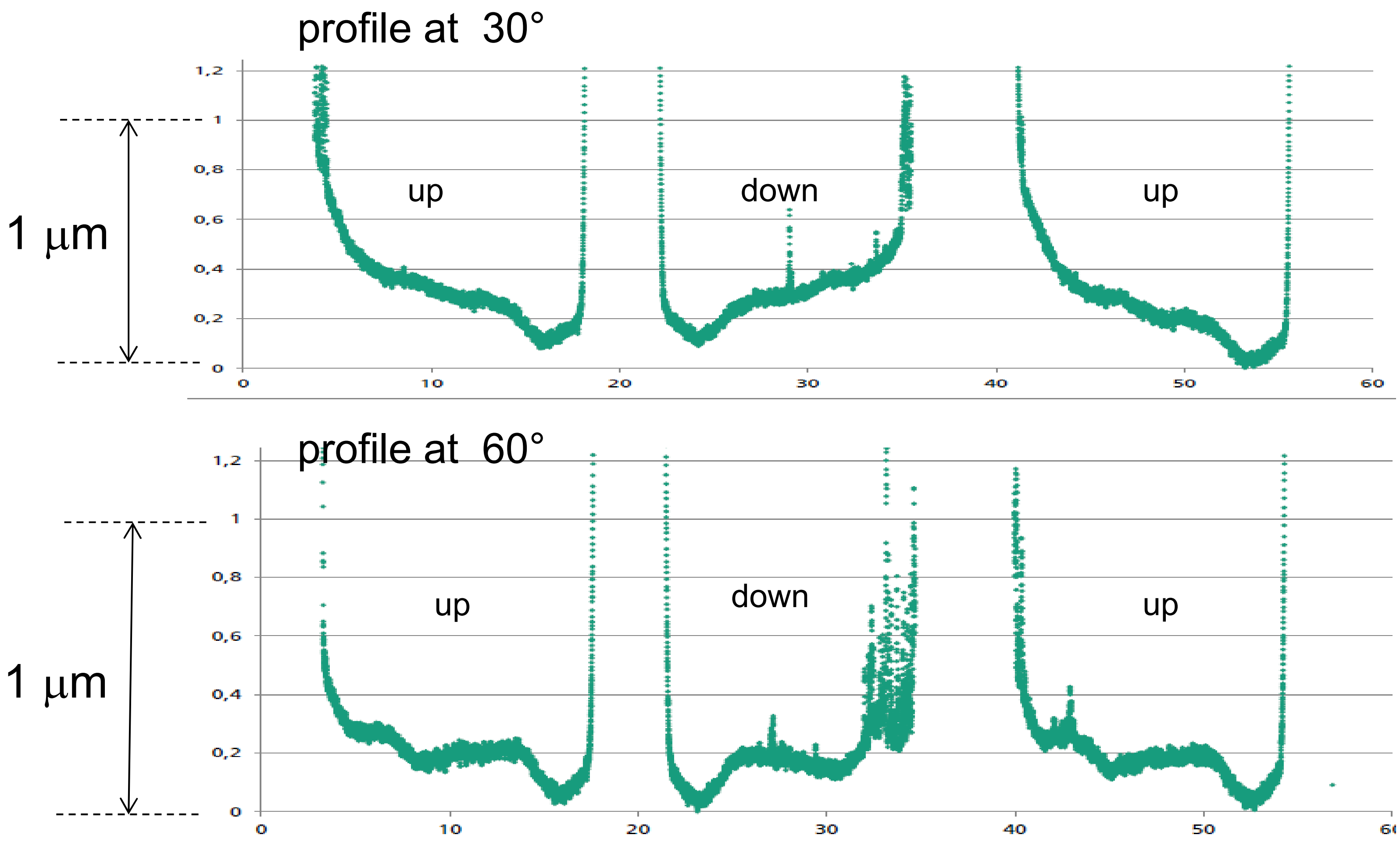
Measurement setup for gap detection (assembly without magnets)



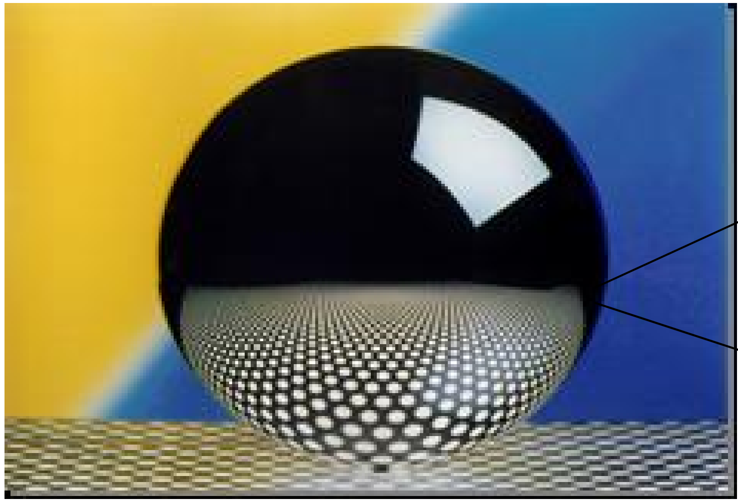
2 capacitive sensors in opposition

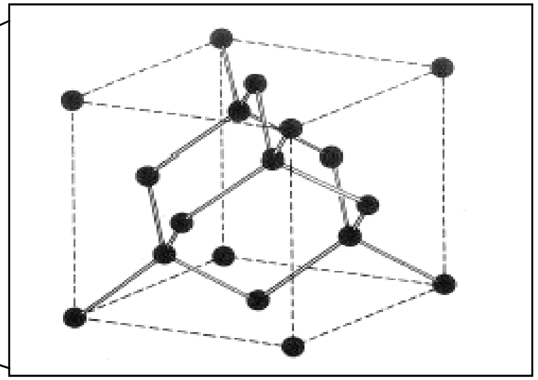


BIPM watt balance: magnet verification



X-ray crystal density technique (XRCD)





8 atoms
per unit cell

$$M = N\,\overline{m}_{Si} = \frac{8V}{a_0^3}\,\overline{m}_{\mathrm{Si}}$$

Measurands

- volume of the sphere
- mass of the sphere
- lattice constant
- mean molar mass

o crystal imperfections
o surface layers

result in 2005: u_r = 3 x 10^{-7}

repeated with isotopically pure ^{28}Si (99.995 %)

result in 2011: u_r = 3 x 10^{-8}

Target for 2014: 2 x 10^{-8}
2015: 1.5 x 10^{-8}

X-ray crystal density technique (XRCD

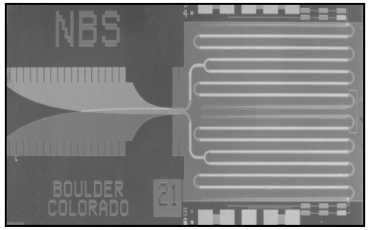
Session - “Metrology for SI”
14:15 to 15:15
Room K7

On the possible future revision of the SI

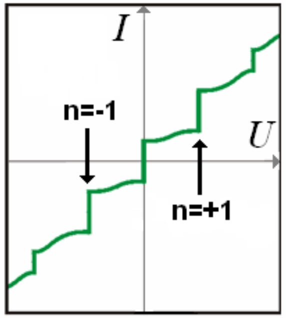
- Why change?
  - Mass (kilogram)
  - **Electricity (ampere)**
  - Temperature (kelvin)
  - Amount of substance (mole)
- What happens next?

Macroscopic quantum effects as the basis for the reproduction of the electrical units

Josephson effect



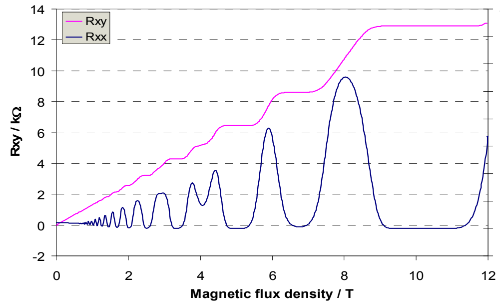
NIST / Wikimedia Commons

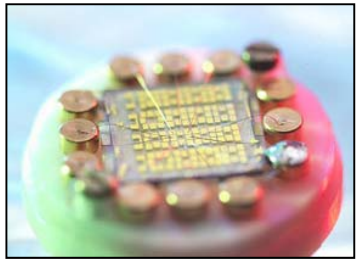


$$U_J = n\frac{f}{K_J}, \qquad K_J = \frac{2e}{h}$$

~~$K_{J\text{-}90} \equiv$ 483 597.9 GHz/V~~

Quantum-Hall effect





$$R_H(i) = \frac{R_K}{i}, \qquad R_K = \frac{h}{e^2}$$

~~$R_{K\text{-}90} \equiv$ 25 812.807 Ω~~

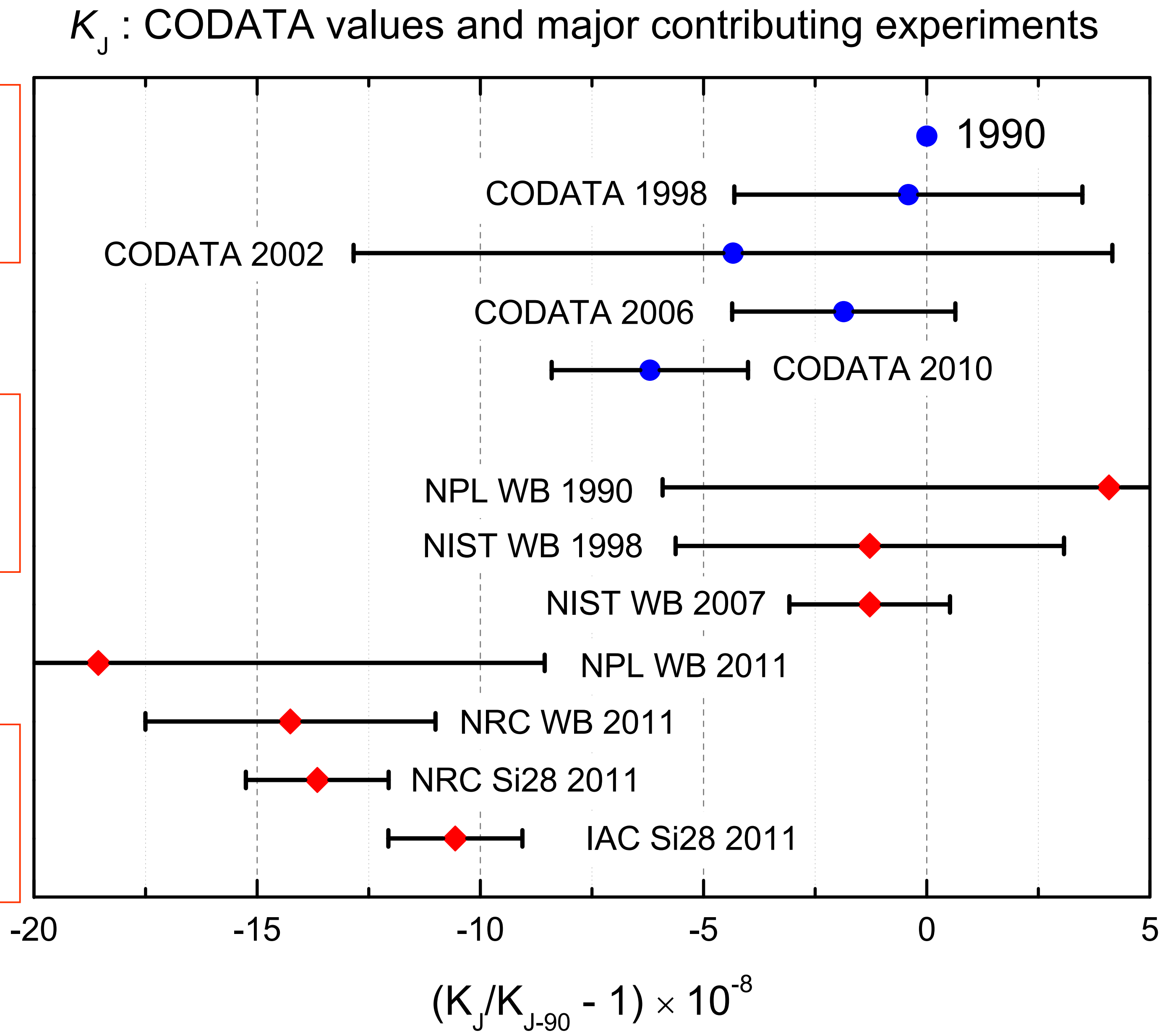
- **Excellent reproducibility underpins the worldwide uniformity of electrical units**
- **But: not within the SI ($\mu_0 \equiv 4\,\pi\,10^{-7}$ N A^{-2}) because "conventional values" $K_{J\text{-}90}$ and $R_{K\text{-}90}$ were adopted in 1990.**
- **Proposal for the new SI: fix the numerical value of *h* (for the kilogram) and the numerical value of *e* (for the ampere).**

Consequences for electrical measurements

$$K_J = \frac{2e}{h}$$

$$R_K = \frac{h}{e^2}$$

$$\left(R_K\, K_J^2\right)^{-1} = \frac{h}{4}$$



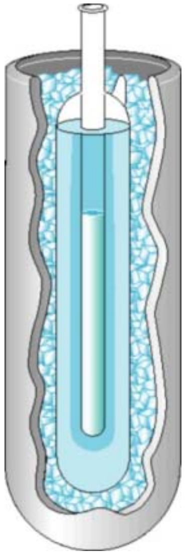
On the possible future revision of the SI

- Why change?
  - Mass (kilogram)
  - Electricity (ampere)
  - **Temperature (kelvin)**
  - Amount of substance (mole)
- What happens next?

The base unit of temperature - kelvin

The 1954 definition

T_{TPW} = 273.16 K



Some limitations

- **Defines only one temperature,**
- **Based on uncontaminated(?) water, and a specified isotopic content,**
- **Influenced by: gradients, annealing etc.**

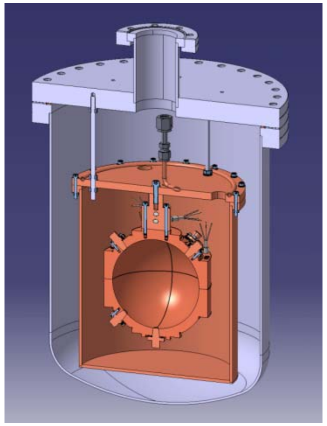
If an energy *E* is measured at a thermodynamic temperature *T* and if *E* is described by a function *f* (*kT*)

- **At present, *k* is determined from $E = f(kT_{TPW})$: T_{TPW} is exact.**
- **In the new SI, *T* measured from $E = f(kT)$: *k* is exact.**

Note: The ITS-90 is decoupled from the present definition of the kelvin and can still be used after the redefinition with the same uncertainty

New measurements of the Boltzmann constant

- **Acoustic Gas Thermometry (NPL, LNE-INM, INRIM, CEM, NIM…)**
- **Dielectric Constant Gas Thermometry (PTB, …)**
- **Johnson Noise Thermometry (NIST,…)**
- **Doppler-Broadening Thermometry (Univ. Paris N./LNE-INM, DFM,…)**



- **The Consultative Committee for Thermometry (CCT) recommends:**
  - **achieve an uncertainty of ≤ 1 ppm in k (~0.3 mK at T_{TPW}), ideally with confirmation by different methods.**
  - **It seems that this goal is well within reach.**

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  - Mass (kilogram)
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  - Temperature (kelvin)
  - **Amount of substance (mole)**
- What happens next?

The mole – based on the Avogadro constant

1971

– "The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogramme of carbon 12.

When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, or other particles, or specified groups of such particles".

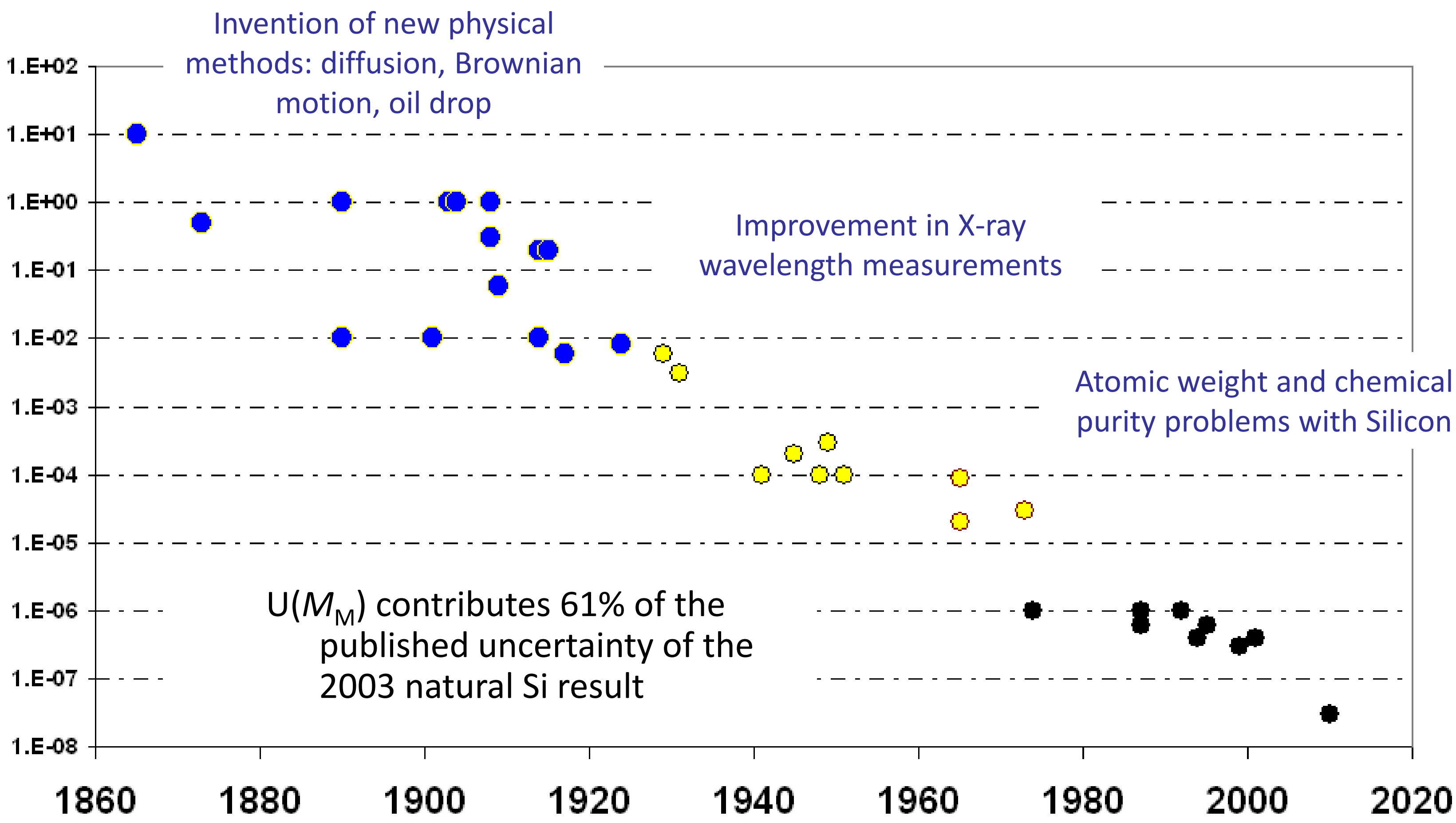
201X

– "The mole is the unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be equal to exactly 6.022 14X 10^{23} when it is expressed in the unit mol^{-1}."

The mole – based on the Avogadro constant

- The proposed new definition of the mole would "reverse" the present definition
  - specify the number of entities in one mole
    - *equal to* N_A exactly.
  - aAdd some uncertainty in the mass of one mole
    - one mole of carbon-12 = 12g +/- $u(a^2)$.
- The molar masses and the atomic masses will have the same (relative) uncertainties.
- A single entity will be an exact amount of substance.
- The old and new definitions will be the same in practice
    - to within +/- $u(a^2)$

The Avogadro constant



Becker, Rep Prog Phys 2001

On the possible future revision of the SI

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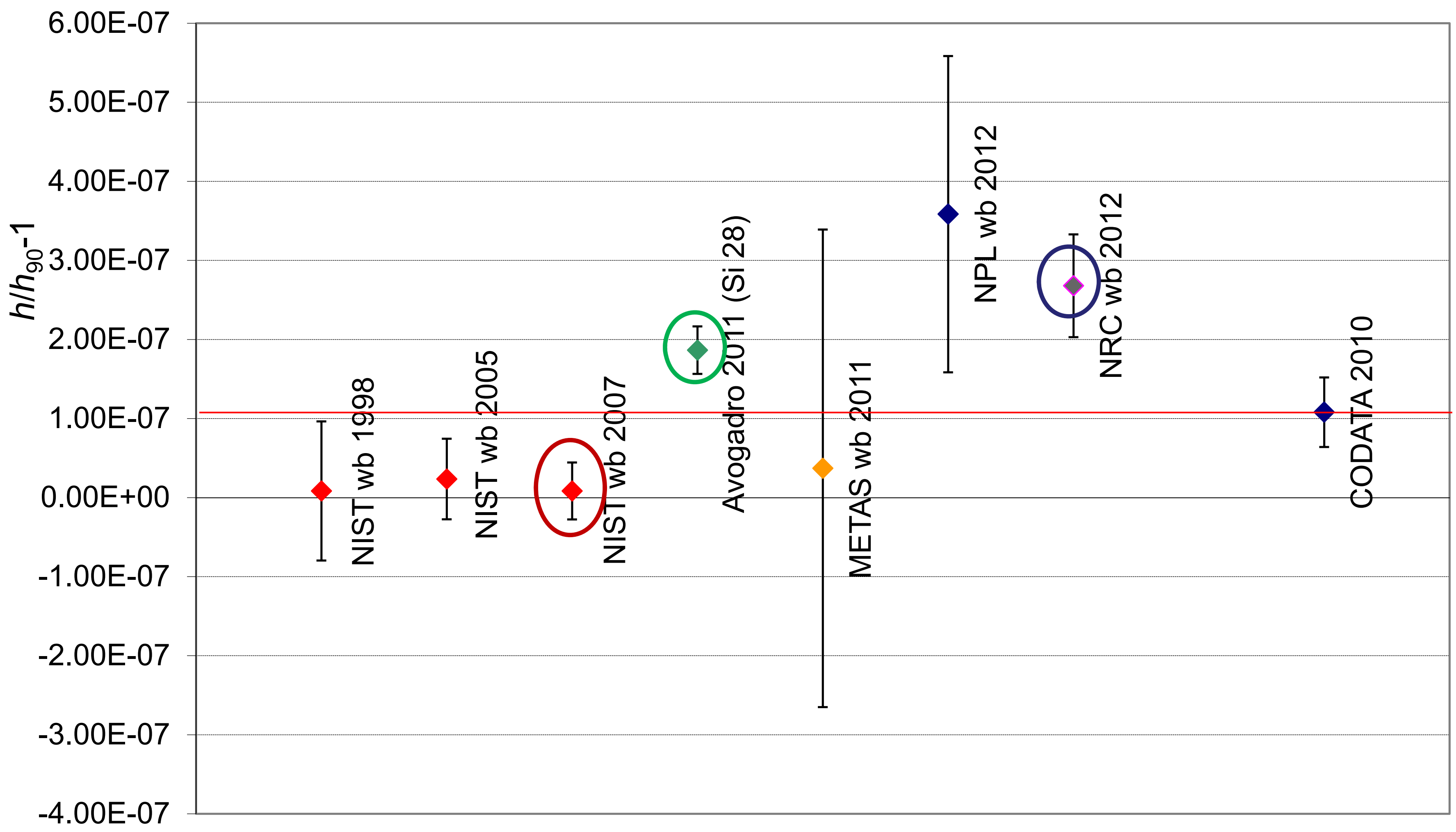
Requirements for the redefinition

In order to ensure that there is no change in the kg as disseminated to users, the CCM (Consultative Committee for Mass) made the following recommendation in 2010 (and confirmed it in February 2013):

1. **Condition on measurements of the Planck constant to be met before redefining the kilogram:**
   i. **at least 3 independent results (eg watt balance and XRCD) with $u_r < 5 \times 10^{-8}$**
   ii. **at least 1 result with $u_r \leq 2 \times 10^{-8}$**
   iii. **results consistent**
2. **Traceability to the IPK of BIPM working standards and of mass standards used to determine *h* needs to be re-established ("Extraordinary Calibrations")**
3. **A *mise-en-pratique* for the definition of the kilogram to be agreed.**

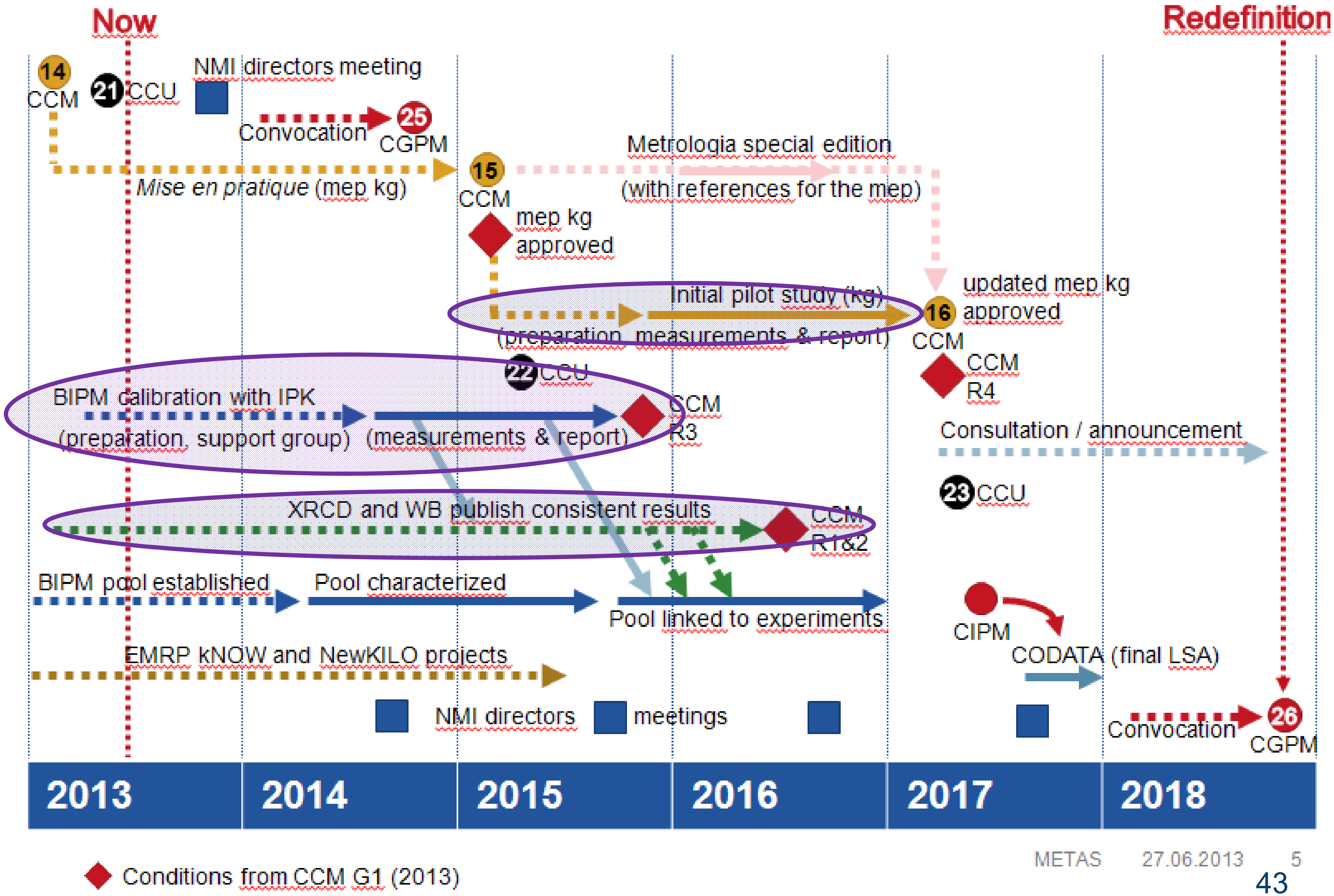
Results for the Planck constant – until last week



Expected results for the Planck constant

Avogadro collaboration (^{28}Si-sphere),	u_r close to 2 x 10^{-8} aimed at for 2014 u_r close to 1.5 x 10^{-8} aimed at for 2015
NIST, NRC watt balances	joint effort to resolve the difference
NRC watt balance	expect 2.5 x 10^{-8} in 2014
LNE watt balance	first measurements mid 2012, objective u_r close to 7.5 x 10^{-8} in 2014
METAS watt balance	new instrument being developed, measurements 2013-2014, target 1 x 10^{-8}
BIPM watt balance	first meas. made, $u_r \leq 5$ x10^{-8} planned for 2018
NIM joule balance	under development, < 10^{-7} in 2019
MSL watt balance	first measurements 2014 but no published results, target 1 x 10^{-8}

The CCM roadmap towards a redefinition in 2018



Source: P. Richard, *The CCM view*, 29th May 2013

Consequences for mass and electrical measurements

Mass metrology:

- mass **values** would not change $m(\text{IPK})_{new} = m(\text{IPK})_{present} \equiv 1$ kg
- mass **uncertainties** would increase $u_r(m(\text{IPK})_{new}) \sim 2 \times 10^{-8}$
  $u_r(m(\text{IPK})_{present}) = 0$

Electrical metrology:

- **values** would change (significantly): $K_J \neq K_{J-90}$, $R_K \neq R_{K-90}$
- **uncertainties** would dissappear: $u_r(2e/h) = 0$ $u_r(h/e^2) = 0$
  $u_r(K_{J-90}) = 4\text{x}10^{-7}$ $u_r(R_{K-90}) = 1\text{x}10^{-7}$

The changes of the electrical units could be significant for users working at the very highest levels of accuracy and will require some explanation and education.

Conclusion

- The principle of the re-definition of four base units was approved in 2011
- It will be based on a redefinition of the kilogram in terms of a fixed numerical value for *h*, and for the redefinition of the ampere, the mole and the kelvin on other constants.
- The Planck constant has been chosen because it is the fundamental constant of quantum physics, and because it will allow the electrical quantum standards to be fully included in the SI.
- The discrepancy in the experimental data for the Planck constant must be resolved.
- A roadmap is being developed to coordinate all technical and awareness activities leading to a redefinition in 2018.